

No. 100

A SERIES OF SIX DISCUSSIONS

144

ON

TRANSPORTATION ENGINEERING.

BY

R.I. JACKSON B.Sc. (ENG.), M.Sc.

"THEORY OF TRAFFIC FLOW".

May/June. 1965.

Forward Planning Branch,  
City Engineer's Department,  
Johannesburg.

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## INTRODUCTION

### 1.1 Traffic Engineering.

The tremendous growth in highway traffic has loosed in our environment a giant which is a good servant but a bad master. Thousands of individually controlled high-speed vehicles, competing for the same road space, have caused serious losses through congestion and casualty.

Traffic engineering is that branch of engineering concerned with taming this giant to achieve free, rapid flow of traffic and the prevention of accidents and their consequent loss of life, personal injury and property damage.

### 1.2 Theory of Traffic Flow.

Traffic involves the complex inter-relationship of 3 elements: (1)

1. Human road-users.
2. Vehicles.
3. Facilities for the accommodation of their movements.

The relationship is complicated by the disparity in the motivations, social attitudes, training, skills and physical limitations of the users; by the mixture of vehicle characteristics in terms of speed, weight, size and acceleration capabilities; and by the differences in the standards of road facilities.

The theory of traffic flow seeks to explain the behaviour of the facility/vehicle/user complex. By establishing the reasons for traffic flow phenomena, the traffic engineer can base his decisions on fact rather than opinion.

### 1.3 Scope of this Paper.

An exhaustive treatment of the theory of traffic flow is beyond the scope of this paper. The type of thinking and work going on in this area will be illustrated by introducing some of the fundamental relationships.

## 2. CAR FOLLOWING THEORY.

### 2.1 Classic Problem.

Probably the most fundamental study in the development of flow theory is the "car following problem". If two cars travel at the same speed and the first vehicle slows down, how does the following vehicle behave?

It is generally known that at any instant, the acceleration of the second car is related to the difference in speeds, and is expressed by the Herman Equation as: (2)

$$\ddot{x}_{m+1} = C(\dot{x}_m - \dot{x}_{m+1})$$

where  $\ddot{x}_{m+1}$  = acceleration of the (m+1)<sup>th</sup> car  
 $\dot{x}_m$  = velocity of the m<sup>th</sup> car  
 $\dot{x}_{m+1}$  = velocity of the (m+1)<sup>th</sup> car  
C = a function of the time-space gap between the vehicles and of the vehicle technology.

Observations of the speed differences for different time spacings between vehicles have shown that when cars are more than 9 seconds apart they have little influence on each other. As the gap reduces below 9 seconds, the following car's speed rapidly approaches that of the leading car.

### 2.1 Stream Shock Wave.

When a stream of cars travels in procession, deceleration of one car is followed by deceleration of the following car, in accordance with the Herman Equation, and delayed by the response time of the following driver. This deceleration is in turn transmitted to the car following him and so on down the stream. This concertina effect travels down the stream like a shock wave.

On one California freeway a dog ran across the freeway, causing a car in the outer lane to brake and was struck by a car in the second lane (marking the spot of the first car's braking). The car following in the outer lane braked and avoided a collision and so on down the line, until 2 miles back one car failed to brake in time, had a rear-end collision and caused a "multi-car pile-up".

An intensive research programme is presently being conducted at the Ohio State University on this shock wave phenomenon, using helicopters with aerial cameras and time lapse mechanisms. Marked cars are being plugged into the stream on freeways to cause the shock waves.

FLOW MEASUREMENTS.

3.1 Speed Characteristics.

The maximum speed potential of automobiles has been steadily rising. The average 1941 model was capable of 86 miles per hour, while the average 1955 model could do 97 miles per hour.

Average speed driven on roads is also rising although slowly. The United States national average in 1941 was 47.1 miles per hour and in 1957 was 50.8 miles per hour. However, locations with average speeds over 60 miles per hour are today more common than those over 50 miles per hour in 1941.

	Average m.p.h. U.S.	% Over 60 m.p.h.
Passenger Cars	52.0	16
Trucks	46.6	3
Buses	53.2	16

On the micro-scale, the spot-speeds of vehicles, passing a point on a road usually follow a Normal Distribution, or are fairly close to a normal distribution. The "standard deviation" measures the dispersion of the speeds about the average speed. From an accident hazard point of view, the standard deviation of the speeds is more significant, generally, than the average speed. If there is a large standard deviation, then there will be a lot of overtaking, lane-changing and braking.

In a recent prize-winning paper, William C. Taylor reported that there was a relationship between the rate of accidents and the distribution of speed on a road (3). The co-efficient of skewness is a measure of the distribution's normality. He showed that where speed zoning (i.e. enforcing speed limits) has the effect

of changing/ ... ..

of changing the distribution from non-normal to normal, as determined by the skewness co-efficient, there is a significant accident reduction. This research suggests a new method of deciding speed limits.

### 3.2 Density.

Density is a measure of the concentration of traffic on a road. It is measured in vehicles per mile. In a traffic jam, when cars are bumper to bumper, the density is at its maximum, even though there is no movement of traffic. Obviously, the higher the density, the more restricted cars are in their movements and the slower they must travel to avoid a rear-end collision due to unexpected deceleration by the vehicle in front.

Some figures are given below to illustrate the order of densities occurring:

Maximum (bumper to bumper) = 300 vehicles per mile.

At expected maximum volume = 60 vehicles per mile.

John C. Lodge Freeway (at 55 m.p.h.) = 36 vehicles per mile.

### 3.3 Volume.

Volume is a measure of the amount of traffic flowing on a road. It is measured in vehicles per hour (or vehicles per minute for certain specific applications).

No mathematical models have been developed for expressing volumes by hour of day, but for any specific location the pattern by hour of day, day of week, week of month and month of year is surprisingly stable.

For design purposes the peak 5-minute volume is an important measurement. Generally, the 5-minute volumes within the peak hour follow a Poisson Distribution.

The highest lane volume ever recorded is 2,437 vehicles in one hour on a freeway in California!

Other high volumes reported are:

4 lanes/ ... ..

4 lanes in one direction - 3,526 - 4,550 vehicles per hour.  
6 lanes in one direction - 5,268 - 6,630 vehicles per hour.  
8 lanes in one direction - 6,883 - 7,964 vehicles per hour.

### 3.4 Capacity.

Possible Capacity of a lane or roadway is the maximum number of vehicles that can pass a given point per hour under prevailing roadway and traffic conditions. It is that volume of traffic which cannot be exceeded without changing one or more of the conditions which prevail (4).

Roadway conditions reducing the possible capacity below optimum value are:

1. Lane width less than 12 feet.
2. Vertical obstructions, such as retaining walls, poles, parked cars, less than 6 feet from the edge of the nearest lane.
3. Absence or inadequacy of shoulders.
4. Lack of sight distance.
5. Long or steep grades.

Traffic conditions affecting the capacity are:

1. Proportion of trucks.
2. Operating speed.

Practical capacity is defined as the maximum number of vehicles passing a point per hour without traffic density being so great as to cause unreasonable delay, hazard or restriction to the driver's freedom to manoeuvre, under prevailing roadway and traffic conditions.

Basic Capacity is the maximum volume under the most nearly ideal roadway and traffic conditions.

The Highway Capacity Manual lists for multi-lane highways: (4).

Basic/ ... ..



Basic capacity - 2,000 passenger cars per lane per hour.  
Practical capacity  
(urban) .. .. - 1,500 passenger cars per lane per hour.  
Practical capacity  
(rural) .. .. - 1,000 passenger cars per lane per hour.  
Urban conditions presume speeds 35 - 40 miles per hour.  
Rural conditions presume speeds 45 - 50 miles per hour.

#### 4. CONGESTION THEORY.

##### 4.1 Density vs. Speed.

The greater the density (i.e. the closer cars are to each other in a traffic stream) the slower they travel, instinctively adjusting this speed to be able to avoid a rear-end collision with the car in front should it suddenly decelerate.

##### 4.2 Volume vs. Speed.

When cars are 9 seconds apart they have little influence on each other's speed. This corresponds to 400 vehicles per hour. As the volume increases they become more and more restricted by the vehicles ahead and the average speed is reduced. The speed reduction continues fairly uniformly with volume increase until the capacity of the road is reached and then a sudden sharp reduction occurs and this is followed by a rapid reduction in the volume of vehicles able to pass along the road. The situation can deteriorate until zero speed and zero volume occur. This is the traffic jam!

##### 4.3 Volume vs. Density.

At low densities, regardless of how high the speed, the volume passing will obviously be low. As the number of cars per mile per lane on the road increases, the volume passing also increases, until the density reaches approximately 60 cars per mile. Above this density the restriction reduces the speed and the density becomes greater without increasing the volume. This volume is the capacity. Beyond this point, the density continues to increase and the volume decreases to zero. This, again is the traffic jam.

Generally/ ... ..

Generally, the relationship between volume, density and speed can be expressed by the equation:

$$\text{Volume} = \text{Density} \times \text{speed}.$$

#### 4.4 Congestion and Ingression Losses (with example).

Congestion time losses are those losses resulting from reduction in the "free flow" velocity by other units sharing the system, i.e. caused by the prevailing volume.

Ingression losses are incurred by units waiting for entry onto the path, when the dead-line demand exceeds some "choke-capacity", such as a bottle-neck on road due to road narrowing, or entrance to a road from a parking garage.

Congestion and ingression losses are illustrated in figure 1 below.

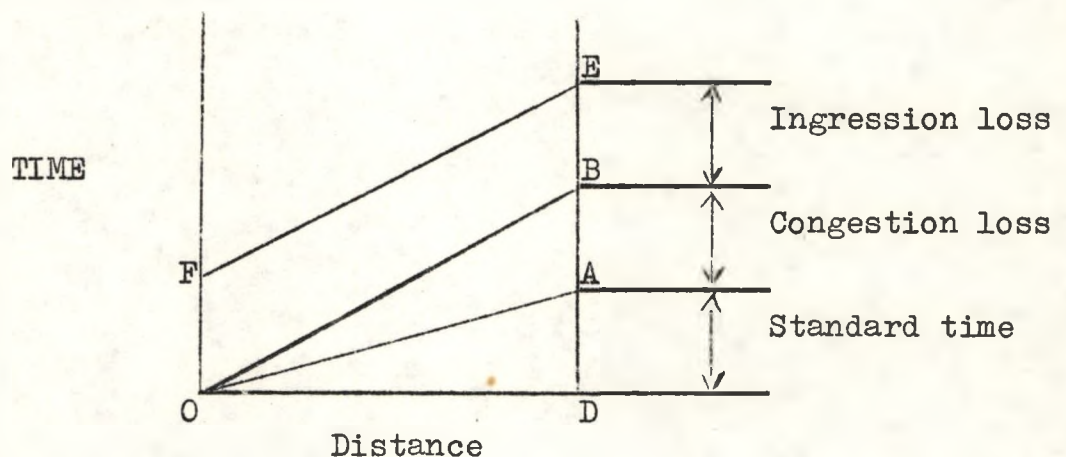


Figure 1. Motion Diagram.

In Figure 1, the motion diagram, OD is the distance travelled.

AD is the time to overcome this distance at a standard speed set by policy.

Slope OA represents the standard speed.

Slope OB represents the speed due to congestion.

B is the time the first element arrives at the destination, and AB is the time lost due to congestion.

E is the time the last element arrives and EB is the ingression time loss.

The/ ... ..

The mathematics of system ingression is amongst the most interesting in traffic theory. Development of the theory is beyond the scope of this discussion.

An example of a simple capacity choke, causing pure ingression is given below:

EXAMPLE: A road has a capacity of 2,000 vehicles per hour, and there is a simple dead-line demand of 1,000 vehicles (say, at a parking garage). Vehicles can be evacuated from the terminal at a maximum of 1,200 vehicles per hour. Compute the ingression level.

$$\text{Total ingression, } Y = \frac{N^2}{2C} \quad (5)$$

where  $Y$  = ingression loss (hours)  
 $N$  = dead-line demand (vehicles)  
 $C$  = capacity (vehicles per hour)

If there were no capacity choke (at the terminal) and path capacity dominated,

$$Y_p = \frac{1000^2}{4000} = 250 \text{ hours}$$

However, the terminal capacity choke reduces this to a simple system with a capacity of 1,200 vehicles per hour,

$$\text{and } Y_c = \frac{1000^2}{2400} = 416.67 \text{ hours}$$

This type of choke capacity occurs frequently along roads, for example at robots or bottlenecks due to a road narrowing.

## 5. PROBABILITY THEORY IN TRAFFIC ENGINEERING.

### 5.1 Distributions.

The variation in the number of vehicles arriving per unit time, at some point on a road is a random phenomenon.

For/ ... ..

For light traffic (well below capacity) the distribution is considered purely random and follows a Poisson Distribution.

Very heavy traffic (close to capacity) can be considered, for design purposes, as regularly distributed, although it does not strictly follow a Normal Distribution, with a small standard deviation.

Intermediate, between random and regular follows a Greenberg or Generalised Poisson Distribution. This is similar to Poisson but has a co-efficient of randomness in the power of the exponential function. When this co-efficient,  $K = 0$ , we have Regular distribution and when  $K = 1$ , we have Poisson.

Frequency distribution theory is very important in modern traffic flow theory as most flow phenomena are being explained in terms of "stochastic" or "probability" models.

A few practical examples of the use of distributions will be given.

## 5.2 Length of Right-turn Slot lane.

Slot lanes in the median island at an intersection are designed so that the number of vehicles desiring to make a right turn will exceed the capacity of the slot lane only a certain percentage of the time. (6).

EXAMPLE: What is the required storage to accommodate 300 right turns per hour, if the slot lane capacity is not to be exceeded more than 5% of the time? The signal cycle is 60 seconds.

Let  $m$  = average number of cars turning right per cycle.  
 $t$  = cycle length.  
 $K$  = capacity of the slot lane.  
 $x$  = number of right-turning cars in any cycle length.

$$\text{Then, } m = \frac{300}{60} = 5$$

P/ ... ..

$$P(x \leq K) = \sum_{x=0}^K \frac{m^x e^{-m}}{x!} \quad (\text{Poisson})$$

Hence,

$$0.95 = \frac{1}{0!} 5^0 e^{-5} + \frac{1}{1!} 5^1 e^{-5} + \dots + \frac{1}{K!} 5^k e^{-5}$$

Terms on the right hand side are developed until the equation is satisfied. Using Poisson tables (7) we find

$$\text{For } K = 9, P(x \leq k) = 0.9682$$

The storage capacity should be 9 vehicles.

### 5.3 Reservoir Space for Parking Garage Entrances.

The arrival rate of cars at a parking garage follows a Poisson Distribution. The rate of servicing (i.e. acceptance and parking, either by ramp or mechanical elevator) also follows a Poisson Distribution. If the average servicing rate equals the average arrival rate, the arriving cars can all be serviced in say, an hour. There will be times, however, when the servicing rate will be below average and the arrival rate above average. Reservoir space is required to store this accumulation.

Fairly sophisticated probability techniques are available for estimating a storage space which would not be exceeded, say, more than 5% of the time.

A useful tool for testing the designed space is the Monte Carlo Simulation technique.

### 5.4 Designing Signal Settings.

The design of signal phase settings is no longer a matter for arbitrary decision. High volume intersections on arterial roads require special turning phases as well as through traffic phases. To cope with the variation in volumes during the day it is common to use traffic-actuated signals. In Britain,

6 out of 7 "traffic lights" are vehicle actuated (8). As many as 8 different phases can occur on a modern high volume cross-intersection.

When cars are halted at a blockade, such as a traffic signal, response time of the first 5 arrivals loses 3.4 seconds more than any subsequent 5. Hence the more cycles there are in an hour, the more time is lost in starting, and the lower the capacity. However, it has been demonstrated that as the cycle length increases, the average delay per vehicle also increases.

A model recently developed showed the delay on a saturated approach is given by: (9)

$$d = \frac{[(1 - \lambda)c - L_d]^2}{2c} \frac{m}{(m - \lambda s)}$$

- where d = average delay per vehicle
- $\lambda$  = proportion of effective green time on the approach
- c = cycle length
- $L_d$  = lost time in phase
- m = rate of vehicle starting
- s = saturation flow for the approach

The optimum cycle length is the minimum cycle providing the required capacity.

The capacity of an intersection with 4 approaches is given by:

$$C = \frac{3600 (c - nL_d) s}{c}$$

- where C = capacity
- c = cycle length
- n = number of phases per cycle
- $L_d$  = lost time per phase
- s = saturation flow.

For arterial volumes below capacity a probabilistic approach is used, permitting a variable cycle, capable of passing at its maximum all the traffic arriving during one cycle 90% of the time, based on a Poisson, or Generalised Poisson distribution.

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